A predictive fabric model for membrane structure design

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Abstract

A predictive model has been developed to determine the biaxial stress-strain response of architectural fabrics, without the need for biaxial testing. Sawtooth and sinusoid models of the fabric unit cell have been formulated, with spring elements between crossovers used to represent the coating. In both models a constant yarn cross-sectional area has been maintained, resulting in a relationship between unit cell length and yarn thickness which eliminates the need to determine the yarn crushing stiffness. A state-of-the-art biaxial test rig and new test protocol have been developed to fully ascertain the stress-strain behaviour of structural fabrics. This enables meaningful comparison to be made between the model output and actual fabric response. The model provides a more accurate representation of fabric behaviour than current industry best practice (i.e. use of elastic constants based on biaxial test data), but without the need for specialist testing or equipment.

Introduction

Coated woven fabrics are used in state-of-the-art structures and yet broad assumptions are made in both material testing and behaviour. The design of fabric structures is hindered by the complex response of coated woven fabrics to biaxial loads in the plane of the fabric. Architectural fabrics have different mechanical properties due to variations in material properties and weave geometry (yarn diameter, weave pattern and coating thickness). Variability in the manufacturing process leads to inconsistencies in properties between fabric batches, and even across the width of a single roll. Biaxial testing is frequently carried out at prestress to determine compensation values, but rarely at working loads to determine fabric stress-strain behaviour for structural design.

Uniaxial strip tests are routinely carried out by manufacturers to determine the ultimate tensile strength of a fabric in warp and fill (or weft) directions. However, measurements of load and extension from these tests give limited information about the biaxial stress-strain behaviour of the fabric. The interaction of warp and fill yarns (crimp interchange, Figure 1) results in complex, non-linear biaxial behaviour that cannot directly be inferred from uniaxial results alone. Despite considerable work in the field, predictive models based on constituent material properties and fabric geometry have so far failed to determine fabric response sufficiently accurately for use in structural design[7,10,15].

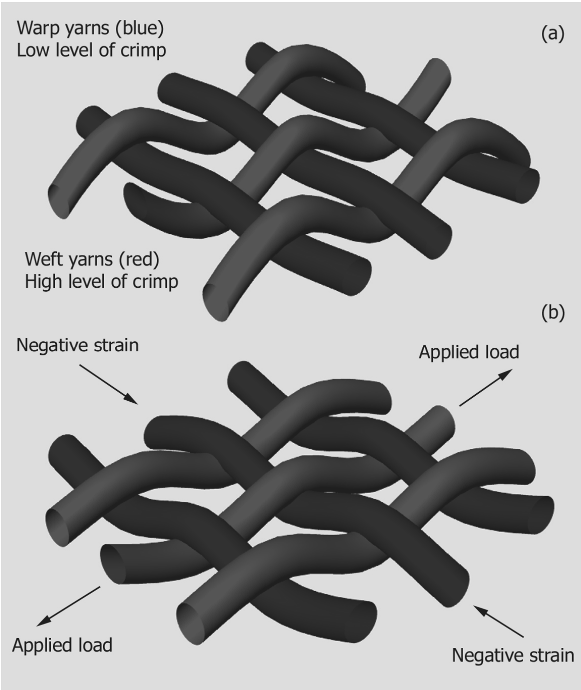


Fig. 1. Crimp interchange mechanism

Aims

To develop a model to predict the in-plane stress-strain characteristics of coated woven fabrics under biaxial load, suitable for use in membrane structure design. The model must be truly predictive with no adjustment of parameters required to fit the output to a given data set. The model should be valid for a wide range of fabrics. It should be easily accessible to the design engineer, with input parameters which can be measured using standard tests and/or commonly available equipment, with no specialist software or computer hardware required to run the model.

Biaxial testing

To assess the validity of a predictive model it is essential to have comprehensive test data with which to compare the model output. Biaxial tensile testing of a cruciform specimen with slit arms[2] has been carried out using a purpose built test rig (Figure 2). A key feature of the test rig is the 'floating frame' concept, developed by Architen-Landrell (Chepstow, UK; www.architen.com). The upper reaction frame is mounted on spherical bearings and is free to move in the plane of the fabric. Due to bowing of the fill (or weft) yarns during weaving and coating, the angle between warp and fill is not necessarily 90° and can vary between 85° and 95° in PTFE-glass fibre fabrics. The cruciform specimens are cut in line with the warp and fill yarns, not necessarily orthogonally. Fabrics will always resist loads along the line of the yarns, hence it is appropriate to cut the samples in this manner. When load is applied to the cruciform the 'floating' upper frame becomes aligned with the cruciform / fabric axes. This allows more accurate measurement of fabric biaxial behaviour without introducing unwanted shear effects. Warp and fill strains are measured using two laser extensometers, one mounted above the cruciform and the other below. The lasers are mounted on the frame such that they follow the fabric centreline position and orientation. Load is applied using hydraulic cylinders and is measured with load cells mounted on the cylinder ends. The reaction frame allows equal and opposite loads to be applied to the specimen in a given direction, whilst only having to control one hydraulic cylinder per axis.

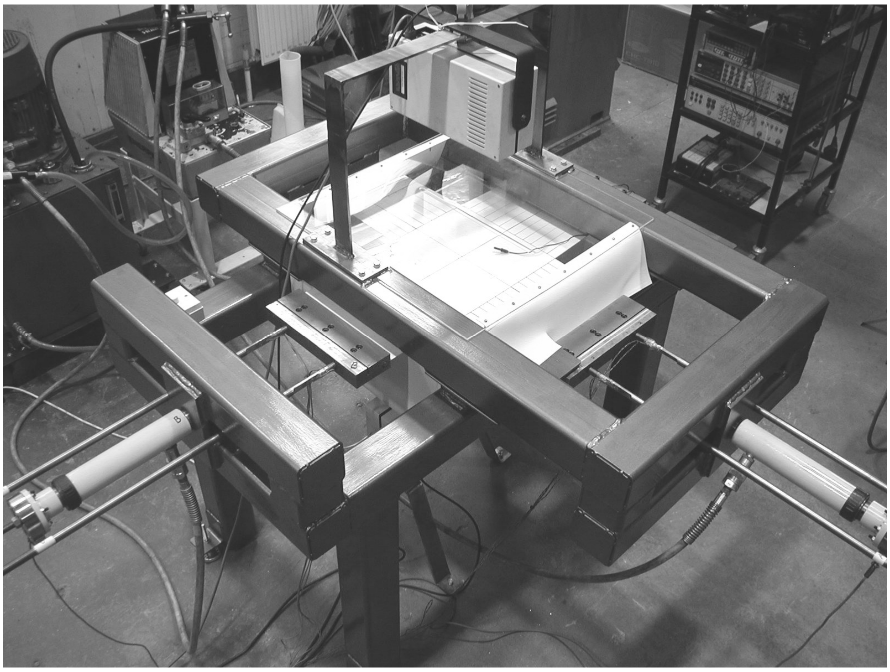


Fig. 2. Biaxial test rig at the University of Newcastle-upon-Tyne

There are currently no British, European or American standards on the biaxial testing of fabrics. Test regimes used in industry are typically devised to inform a 'plane stress' model (i.e. using elastic constants and Poisson's ratios), rather than to fully describe the non-linear fabric response. A new test protocol has been developed based on previous published work, numerical modeling, a review of methods used in industry and extensive testing.

Application of prestress followed by mechanical conditioning provides repeatable stress-strain data suitable for medium to long term structural design. The radial test regime developed for this research (Figure 3) explores all feasible stress states; it is not limited to a few specified stress ratios. A method of removing residual strain from the test data has been developed to prevent skewing of the response surface due to fabric creep.

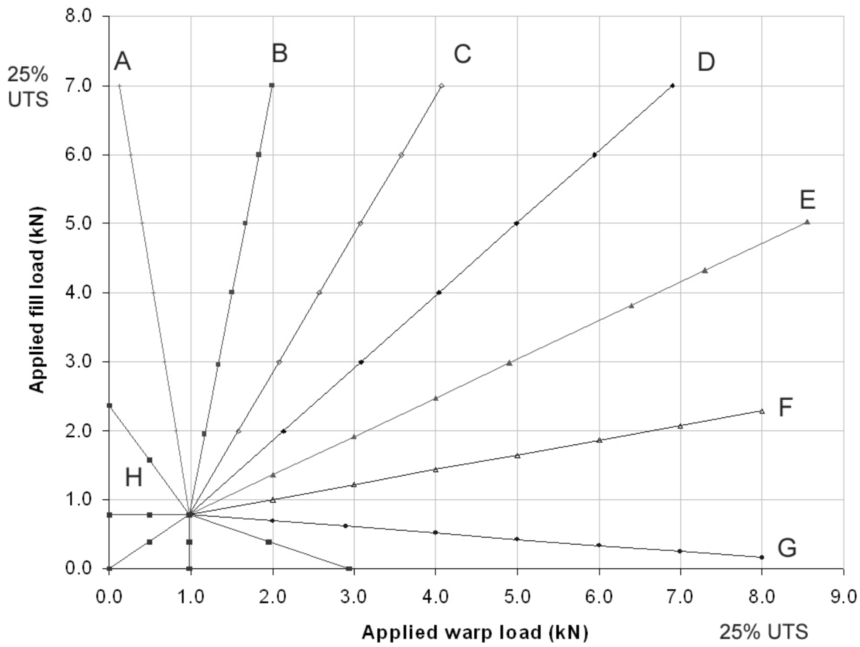


Fig. 3. Biaxial test protocol: radial load regime

This test protocol efficiently populates the stress space with strain data and frequently returns to prestress to enable accurate removal of residual strains. Thorough testing of the effect of load history on each stress state is infeasible; however the loading and unloading results give a good indication of the level of variability. Each radial arm of the load regime is typical of the load paths the structure will experience in a single load event (a gust of wind or snow load), i.e. from prestress to a loaded condition and back to prestress. The high residual strains at the end of a typical test (Figure 4) show the importance of removing residual strain during the test to avoid distortion of the response surface.

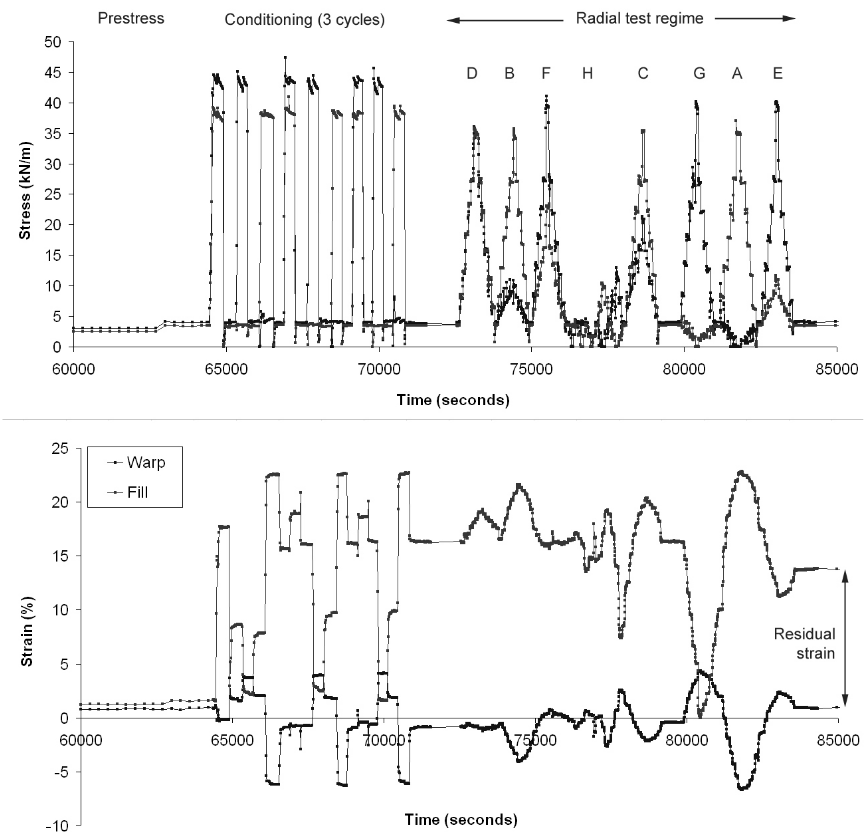


Fig. 4. Load and strain histories for a PTFE coated glass fibre fabric

Predictive model

Sawtooth model formulation

Predictive fabric models typically consider the fabric 'unit cell', the smallest repeated unit in the fabric. For a plain weave fabric this is simply half a wavelength of two intersecting orthogonal yarns.

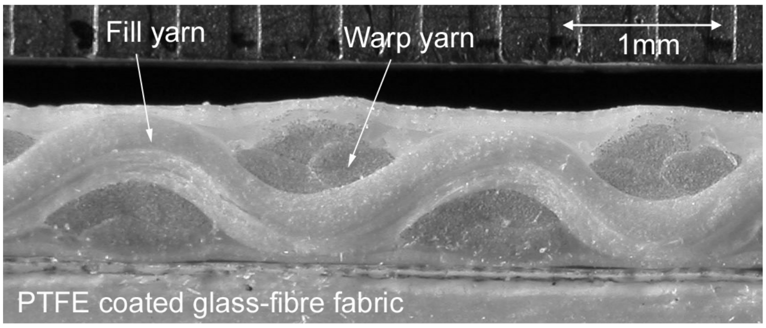


Fig. 5. Sawtooth representation of yarn waveform

A sawtooth model has been developed which includes yarn and coating tensile extension and yarn crushing[8,11]. The model behaviour is elastic; there is no consideration of energy loss and viscoelastic effects. For a given applied load the unit cell geometry is modified to give force equilibrium. The principal constraints are:

1. The sum of the yarn radii must equal the sum of the yarn wave-form amplitudes[11],
2. Assuming negligible yarn bending stiffness, out-of-plane forces must equal zero. For the sawtooth model these out-of-plane forces are a component of the yarn tension at crossovers.

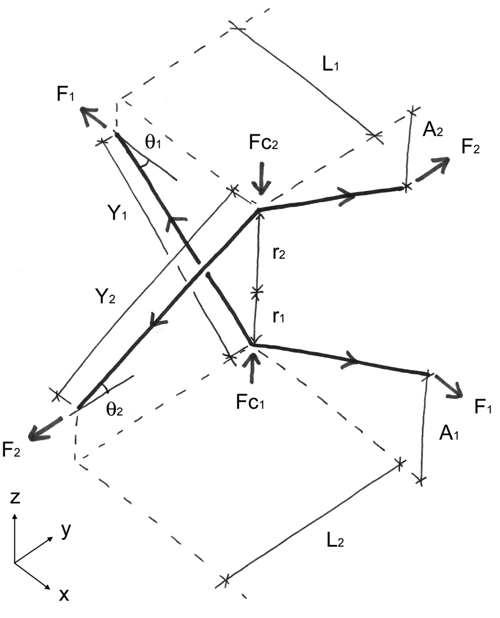


Fig. 6. Fundamental sawtooth unit cell

Determination of the change in yarn radius under load is vital. Previous workers typically use a crushing spring between yarns at the crossovers[4]. However, determination of the value for the spring constant is problematic and requires removal of in-situ yarns and specialist testing[4,12]. Because of this, the yarn crushing stiffness is commonly used as a parameter for calibrating the model against test data, which compromises the predictive nature of the model.

For this work a constant yarn cross sectional area has been adopted[5,6]. As well as obviating the need to define the yarn crushing stiffness, this enables the yarn cross-section to be modelled such that it is consistent with the wave-form of the orthogonal yarn. The unstrained yarn cross-section can be defined by an equilateral parallelogram, or rhombus. As load is applied the rhombus deforms to become a quadrilateral with one line of symmetry, or kite shape (Figure 7). With a constant cross-sectional area, if the yarn length and angle (θ) are known then the radius (i.e. the out-of-plane thickness) of the orthogonal yarn can be calculated.

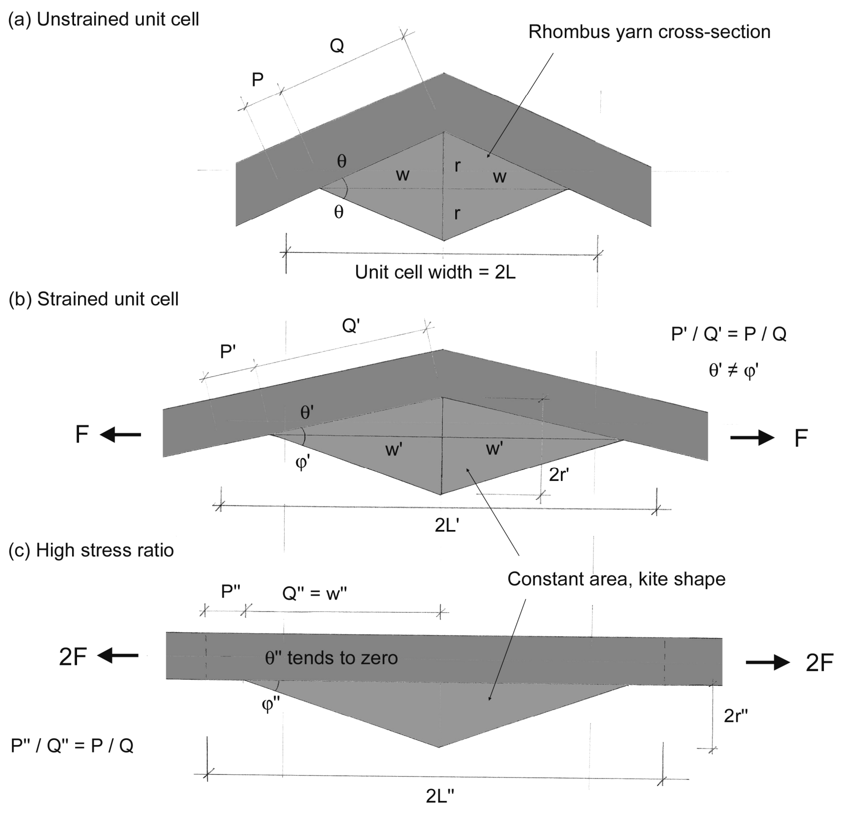


Fig. 7. Rhombus yarn cross section

Sinusoid model formulation

Use of a more realistic representation than the ubiquitous sawtooth may benefit model accuracy[9,15]. Sinusoidal or other curved yarn representations are frequently used in finite element unit cell models, which impose fewer restraints on the geometry. Methods used include:

1. A series of control points interpolated by a Bézier curve with ellipses defining the yarn cross section[6],
2. A sinusoidal waveform[13,15],
3. A model with no a priori assumptions about the yarn wave form, the geometry of the yarn being a function of the applied loads[9].

Fourier analysis has been carried out on points measured along images of in-situ yarns to determine the most appropriate function to represent the yarn waveform. The correlation of the simple sine curve (or fundamental, $a_0 + a_1 \sin x$) is extremely good, with little benefit in using additional harmonics. The mean deviation from the measured points is only 2.5% of the amplitude. To define a simple sine function only requires measurement of the yarn amplitude and wavelength.

The yarn has been modelled as being composed of many pinned bars with vertical forces (F_{vn}) applied at each node and a horizontal force (F) applied at one end (Figure 8).

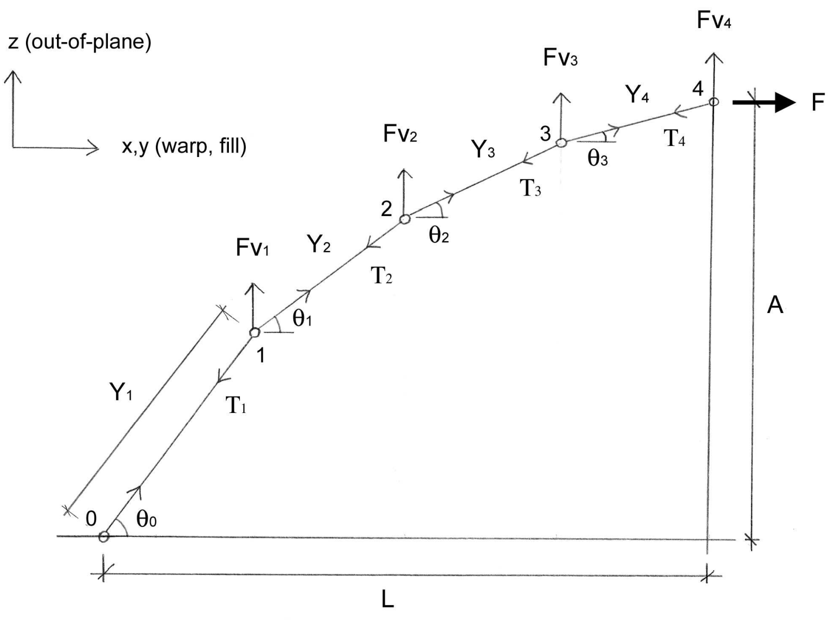


Fig. 8. Sinusoid yarn model

Measurements of fabric geometry provide the amplitude and wavelength, from which the initial shape of the sinusoidal yarn is known. Hence angles θ_n and θ_{n-1}

are known at each node. The tension in the yarn, and the resultant tensile extension, will vary along the length of the yarn. Starting with the known applied load (F), forces can be resolved at node 4, and then at subsequent nodes along the yarn. The distribution of forces varies with the ratio of wavelength to amplitude. This provides a series of contact forces appropriate to the weave shape.

A constant yarn cross-sectional area constraint is used, similar to that used in the sawtooth model but with the yarn cross-section being bounded by two intersecting sinusoids. This gives a more realistic representation of the yarn cross-section, and a correspondingly more accurate calculation of the yarn cross-sectional area (Figure 9). As for the sawtooth model, this formulation provides a geometrically consistent model of the warp and fill yarns (Figure 10), and removes the need to determine the yarn crushing stiffness. An iterative process is used to determine values of warp and fill wavelength and amplitude that provide balanced out-of-plane forces, consistent geometry and constant yarn cross-sectional areas.

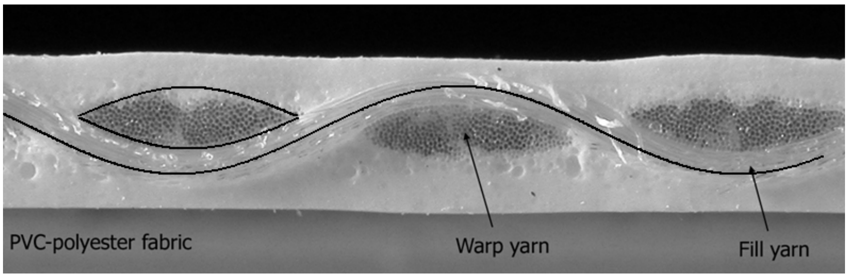


Fig. 9. Sinusoidal representation of yarn cross-section

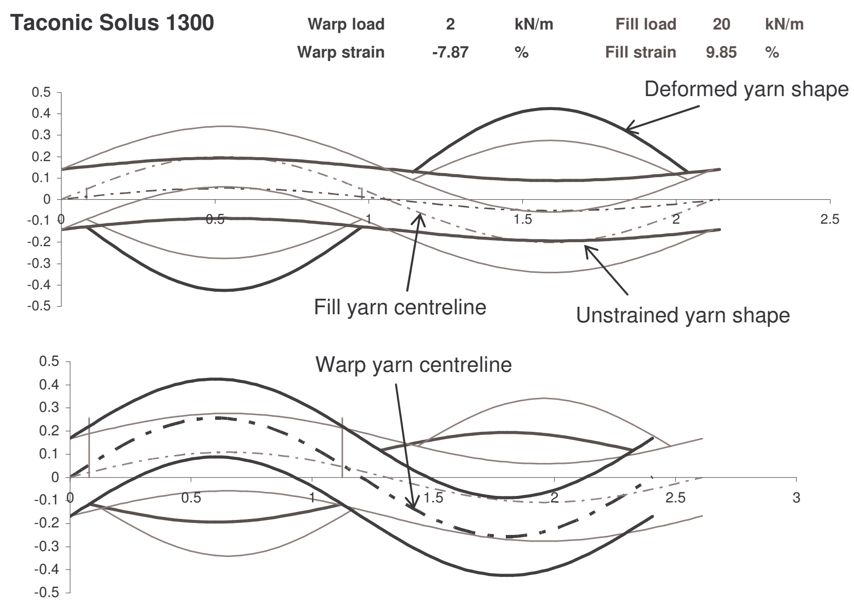


Fig. 10. Sinusoidal model

Model input data

Yarn and coating tensile properties have been evaluated using stress-strain data from standard uniaxial tests[3]. The principle is that the initial part of the stress-strain curve at low load corresponds to coating stiffness, with yarn tensile properties becoming dominant at high loads (Figure 11). This approach has been adopted by several previous researchers[8,14]. This is appropriate for this work which aims to develop a predictive model which does not require specialist testing.

Yarn dimensions and crimp characteristics have been determined using measurements of fabric cross-section images. Recent advances in digital camera technology mean that high quality images of the fabric cross-section can be acquired using an inexpensive digital camera. For this work a Nikon Coolpix 4500 camera was used with a macro lens. Measurements from multiple images were taken and averaged to give typical dimensions for each fabric. The sum of the yarn radii minus the sum of the amplitudes must equal zero for geometric consistency. Measurements were adjusted to ensure consistent measurements were used.

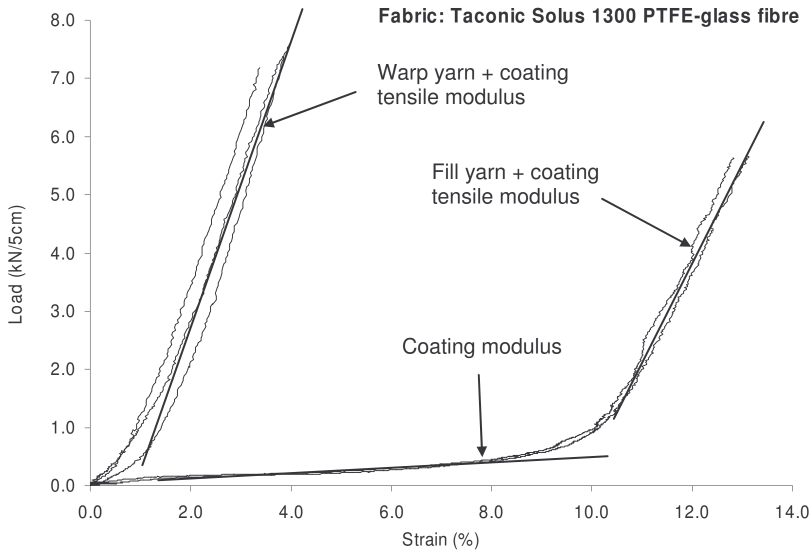
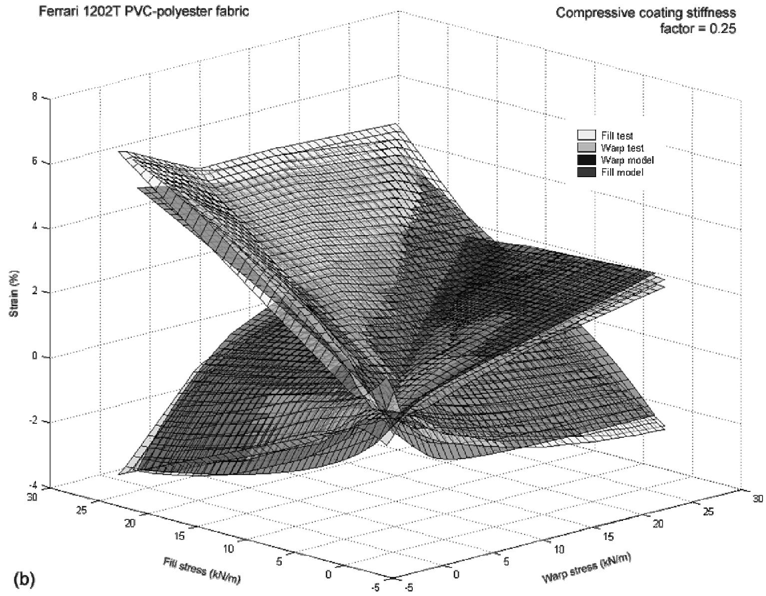


Fig. 11. Uniaxial test data for PTFE-glass fabric

Results and discussion

Biaxial testing has been carried out using a new test protocol which mechanically conditions the fabric before applying a wide range of stress states[1,2]. This provides two response surfaces which fully describe the fabric biaxial stress-strain behaviour, which can be used to assess the quality of the model output. Tests have been carried out on both PVC/polyester and PTFE/glass fabrics from several manufacturers (Taconic, Ferrari and Verseidag) in a range of fabric weights (PVC/polyester, type I-IV/V; PTFE-glass type G5-G7). To enable a direct comparison of the models and test results, the models have been used to calculate strains for the specific loads applied during each test.



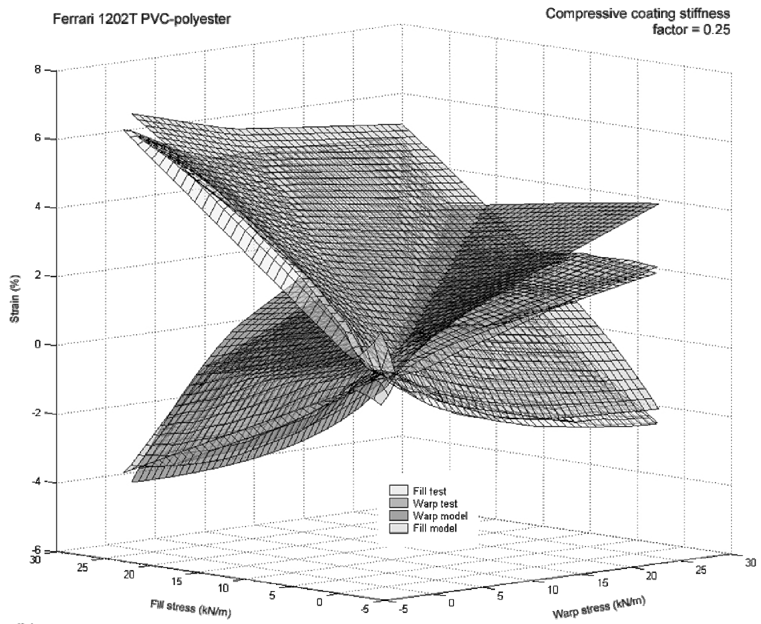


Fig. 12. Sawtooth (a) and sinusoid (b) model output; PVC-polyester fabric

Initially both the sawtooth and sinusoid models predicted excessively large negative strains, particularly at high warp stress and low fill stress. These large negative decrimping strains do not occur in reality because of the compressive resistance of the coating. Because of difficulties in determining the coating compressive stiffness, it has been used as a parameter for calibrating the model against the test data. To make the model generally applicable and ensure that it is truly predictive, the compressive coating stiffness factor must be same for each type of fabric (i.e. for all PTFE-glass fibre fabrics and for all PVC-polyester fabrics) and preferably the same for both types of fabric. As a starting point the compressive coating stiffness was set to be equal to the tensile coating stiffness. The values were then varied to give the best correlation with the test data. It became clear that different values are required for PVC-polyester and PTFE-glass, which is reasonable as the PVC and PTFE coatings have different material properties. The best fit was achieved with a coating stiffness factor of 0.25 for PVC-polyester fabric and 3 for PTFE-glass fibre fabric.

Correlation with the test data is good, particularly for the sawtooth model (Figure 12). No parameters have been varied for particular fabrics, ensuring the model is truly predictive. The model provides a single elastic stress-stress-strain surface, which has been compared with viscoelastic loading and unloading test data, hence a deviation of zero could never be achieved. The deviation of the sawtooth model from the mean of the viscoelastic test data is 5.3 to 5.9% of the strain range. For comparison, the average variation between repeat tests on the same fabric was 3.0% of the strain range. The model output is significantly more accurate than the assumed material properties which are commonly used in industry, and is of a similar level of accuracy to a plane stress representation (using two elastic moduli and Poisson's ratios) based on each set of test data.

Table 1. Comparison of predictive models with biaxial test data

Direction	Sawtooth model		Sinusoid model	
	PVC-polyester	PTFE-glass fibre	PVC-polyester	PTFE-glass fibre
	Root-mean-square deviation of predicted strains from test data; percentage strain (percentage of strain range at design loads)			
Warp	0.31 (7.8)	0.40 (8.1)	0.73 (14.3)	1.07 (14.3)
Fill	0.53 (8.7)	0.72 (7.6)	1.81 (17.5)	2.50 (16.8)

The model output can also be plotted on graphs of strain against the root-mean-square stress along each of the radial load paths[2], providing a good visual assessment of the quality of the model (Figure 13).

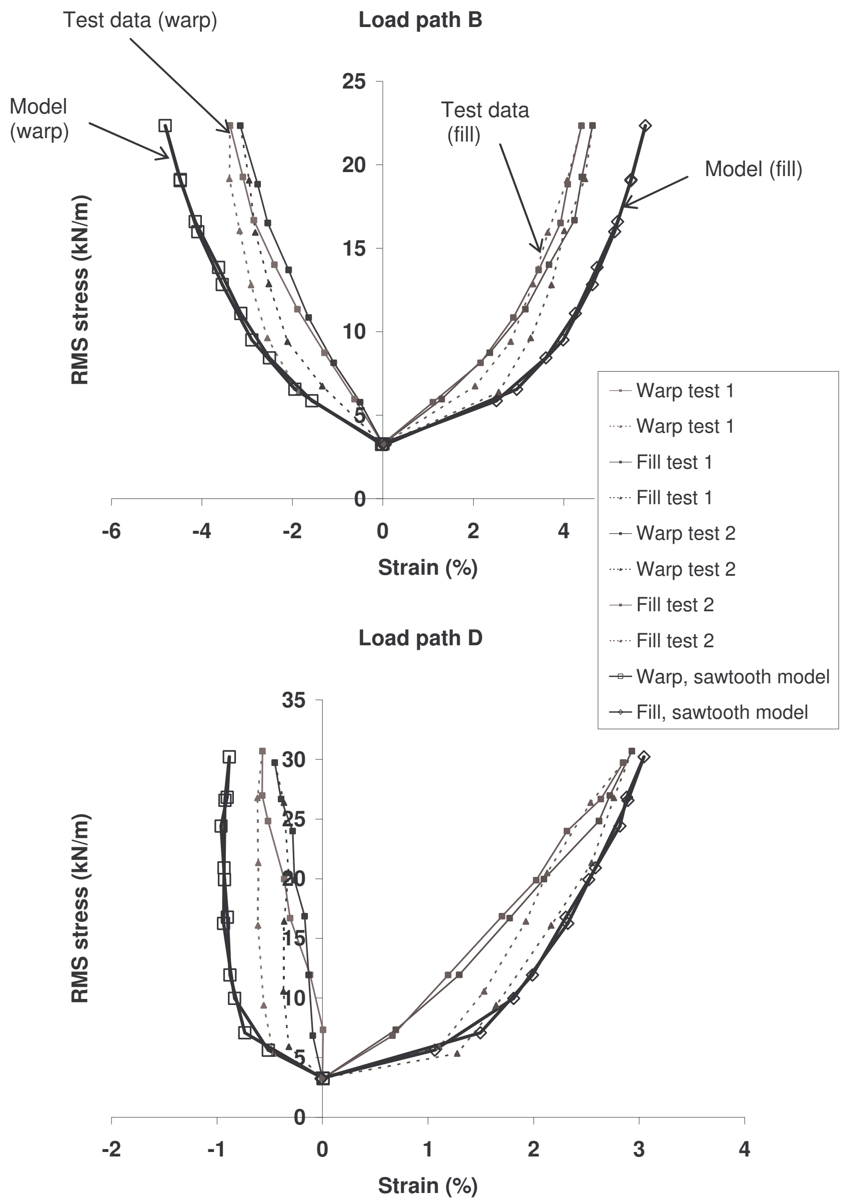


Fig. 13. Comparison along radial load paths, sawtooth model (Verseidag PTFE-glass fibre)

A novel method has been developed for determining crimp equilibrium for a sinusoidal yarn based on contact forces and geometric constraints, which provides a new approach to realistically modelling woven fabrics. Despite this, the sinusoid model as formulated in this work does not provide as good a correlation with the test data as the sawtooth model (Table 1). The yarn length in the sinusoid model is greater for a given yarn geometry than in the sawtooth model. Hence the sinusoid model predicts greater decrimping strains. The difference, between 17 and 20% of the mean value, is clearly significant to the model output. The sawtooth model, giving lower strains with its straight line approximation to the yarn waveform, may be more accurate as a predictive tool as it inadvertently counteracts some of the simplifications in the model. Explicitly, the out-of-plane restraint provided by the coating is not included in the model. At high stress ratios crimp interchange results in a ‘dimpled’ fabric surface with the coating being stretched over the yarns at crossovers (Figure 14). It is postulated here that this effect may be approximated by the shortened yarn length inherent in the sawtooth representation.

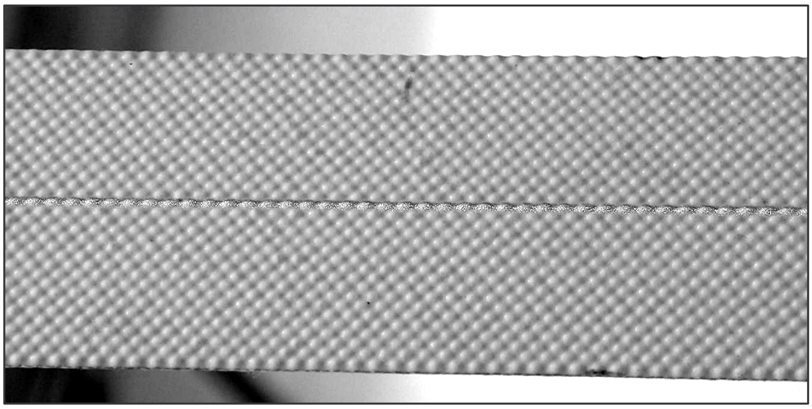


Fig. 14. Fabric surface during uniaxial strip test

Yarn bending stiffness has also been neglected in the model. The large scale bending stiffness of architectural fabrics is generally regarded as negligible, but the yarn bending stiffness may be significant on the scale of the unit cell. Inclusion of these additional factors with the sinusoid formulation may provide an even more accurate model.

Conclusions & applications

A predictive model has been developed to determine the biaxial stress-strain response of architectural fabrics, without the need for biaxial testing. The model provides a more accurate representation of fabric behaviour than current industry best practice (i.e. elastic constants based on biaxial test data), but without specialist testing or equipment. The model is truly predictive; parameters are not optimised to fit the model output to a particular data-set. The model output has been compared with comprehensive biaxial test data for both PTFE-glass fibre and PVC-polyester fabrics.

The model will be particularly for useful for the analysis of small or medium size membrane structures for which comprehensive biaxial testing is prohibitively expensive. Another application is for fabric reverse engineering: if certain stress-strain properties are required for a given application, appropriate yarn properties and weave geometry can be calculated. This can inform the choice of fabric, or enable manufacturers to produce fabrics with particular mechanical properties. This may have benefits in other fields, for example the design of medical textiles which need to replicate the mechanical properties of specific tissues.

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